

Cosmological constraints on Lorentz invariance violation in the neutrino sector

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We derive the Boltzmann equation in the synchronous gauge for massive neutrinos with a deformed dispersion relation. Combining the 7-year WMAP data with lower-redshift measurements of the expansion rate, we give constraints on the deformation parameter and find that the deformation parameter is strong degenerate with the physical dark matter density rather than the neutrino mass. Our results show that there is no evidence for Lorentz invariant violation in the neutrino sector. The ongoing Planck experiment could provide improved constraints on the deformation parameter.

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I. INTRODUCTION

Neutrino oscillations imply that at least two of the three neutrino types have non-zero mass (see Ref. [1] and reference therein). Unfortunately, neutrino oscillation experiments only provide two mass squared differences, but not the overall mass scale. Cosmology provides a promising way to tackle this problem by the gravitational effect of massive neutrinos on the expansion history near the epoch of matter-radiation equality and on the growth of large-scale structures at late time. Precision measurements of the cosmic microwave background (CMB) anisotropies and the large-scale structure distribution of matter have allowed us to determine or constrain the absolute neutrino mass scale [2–5] (see also Ref. [6] for a recent review).

Indeed, neutrino oscillations can be explained by small Lorentz invariance violation even without introducing neutrino mass, as shown in [7, 8, 10–12]. Observed neutrino oscillations may originate from a combination of effects involving neutrino masses and Lorentz invariance violation. The possibilities of Lorentz invariance violation were explored in quantum gravity [13], loop quantum gravity [14], non-commutative field theory [15], and doubly special relativity theory [16].

It is well-known that Lorentz symmetry is a fundamental feature of modern descriptions of the nature, including both the Einstein's general relativity and standard model of particle physics. One can expect that the breaking of Lorentz symmetry may leave some imprints in astrophysical observations such as the CMB anisotropies and large-scale structure of our universe. In this paper, we consider cosmological tests of Lorentz invariance violation in the neutrino sector. Here we focus on the Coleman-Glashow model, in which the energy-

momentum relation is modified by a Lorentz-violating interaction in the framework of conventional quantum field theory [8]. We will build up a Lagrangian and derive the Boltzmann equation in the synchronous gauge for the massive neutrinos with deformed dispersion relation, and then place constraints on Lorentz invariance violation by combining the 7-year WMAP data [3] with the latest distance measurements from the Baryon Acoustic Oscillation (BAO) in the distribution of galaxies [17] and the Hubble constant (H_0) measurement [18].

This paper is organized as follows. In Sec. II we write down the Lagrangian for neutrinos with deformed dispersion relation. In Sec. III we derive the Boltzmann equation for neutrinos in the synchronous gauge. In Sec. IV we place constraints on the deformation parameters using the CMB data in combination with measurements of the Hubble constant H_0 and the BAO feature. Section V is devoted to conclusions.

II. DEFORMED DISPERSION RELATION

At a phenomenological level the deformed dispersion relation for massive neutrinos can be generally parameterized by

$$E^2 = m^2 + p^2 + \sum_{n=1}^{\infty} \alpha_n \frac{p^n}{M^{n-2}}, \quad (1)$$

where E is the neutrino energy, m is the neutrino mass, $p = (p^i p_i)^{1/2}$ the magnitude of the 3-momentum, α_n 's are dimensionless coefficients and M denotes the energy scale corresponding to Lorentz symmetry violation (which is typically taken to be the Planck mass). Such a deformed dispersion relation implies that there are departures from Lorentz invariance in the neutrino sector if $\alpha_n \neq 0$. The $n = 1$ term would produce huge effects at low energy and has been strongly constrained. The p^n term with $n \geq 3$ is suppressed by $1/M^{n-2}$. In the present work we will therefore consider the case of $n = 2$,

$$E^2 = m^2 + p^2 + \xi p^2, \quad (2)$$

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where ξ is the deformation parameter characterizing the size of Lorentz symmetry violation. Such a deformed dispersion relation was constructed by Coleman and Glashow in the framework of conventional quantum field theory [8].

Here we point out that the dispersion relation given in (1) and (2) is not very general. It neglects oscillations, possible species dependence, anisotropies associated with violations of rotation symmetry, and CPT violation. As shown recently by Kostelecky and Mewes, all of these are possible [9]. For example, it is known that taking odd values of n in Eq. (1) corresponds to CPT violation and gives rise to a sign difference for neutrinos and antineutrinos [9]. We have to emphasize that the model considered in this paper is one of many possible Lorentz-violating theories.

The scalar perturbations of the Friedmann-Lemaître-Robertson-Walker metric in the synchronous gauge can be written as

$$ds^2 = a^2(\tau) [-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \quad (3)$$

where $a(\tau)$ is the scale factor, τ is the conformal time, and the scalar mode of h_{ij} is represented by two functions h and η which are defined by

$$h_{ij}(\vec{x}, \tau) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \left[\hat{k}_i \hat{k}_j h(\vec{k}, \tau) + (\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij}) 6\eta(\vec{k}, \tau) \right], \quad (4)$$

in Fourier space, where $\vec{k} = k\hat{k}$. The action for a neutrino with the dispersion relation (2) can be written in the first order formalism by

$$S = \int d\tau [\dot{x}^\mu P_\mu - \lambda (m^2 + g^{\mu\nu} P_\mu P_\nu + \xi g^{ij} P_i P_j)], \quad (5)$$

where λ is a Lagrange multiplier which enforces the mass-shell condition (2) and

$$P_0 = -aE, \quad (6)$$

$$P_i = a(\delta_{ij} + \frac{1}{2}h_{ij})p^j. \quad (7)$$

The above action implies that the Hamiltonian vanishes. Performing the Legendre transformation to the Lagrangian formalism and eliminating λ from the action (5), we obtain the Lagrangian for neutrino as follows

$$L = -ma\sqrt{1 - \frac{u^2}{1+\xi}} \left[1 - \frac{h_{ij}u^i u^j}{2(1+\xi - u^2)} \right], \quad (8)$$

where $u^i = dx^i/d\tau$ is the coordinate velocity and $u^2 = \delta_{ij}u^i u^j$. Following [19] from the Lagrangian (8), the equations of motion in the synchronous gauge become

$$\frac{dx^i}{d\tau} = \frac{(1+\xi)\delta^{ij}q_j}{\epsilon} [1 + \mathcal{O}(h)], \quad (9)$$

$$\frac{dq}{d\tau} = -\frac{1}{2}q\dot{h}_{ij}n^i n^j, \quad (10)$$

where $q_j = qn_j$ is the comoving 3-momentum written in terms of its magnitude and direction with $n^i n_i = \delta_{ij}n^i n^j = 1$, and $\epsilon = \sqrt{m^2 a^2 + (1+\xi)q^2}$ is the comoving energy of neutrinos.

III. BOLTZMANN EQUATION

The number density n_ν , energy density ρ_ν and pressure P_ν for massive neutrinos with (2) are given by

$$n_\nu = \frac{1}{a^4} \int \frac{d^3q}{(2\pi)^3} f_0(q), \quad (11)$$

$$\rho_\nu = \frac{1}{a^4} \int \frac{d^3q}{(2\pi)^3} \epsilon f_0(q), \quad (12)$$

$$P_\nu = \frac{1}{3a^4} \int \frac{d^3q}{(2\pi)^3} \frac{(1+\xi)q^2}{\epsilon} f_0(q). \quad (13)$$

Here the zeroth-order phase space distribution is well approximated by the relativistic Fermi-Dirac distribution $f_0(q) = g_s[1 + \exp(\sqrt{1+\xi}q/T_0)]^{-1}$, where T_0 is the neutrino temperature today and $g_s = 2$ the number of spin degrees of freedom. If $m \gg T_0$, the total mass of neutrinos $\sum m = 94(1+\xi)^{3/2}\Omega_\nu h^2$ eV.

The perturbed energy density, pressure, energy flux, and shear stress in the Fourier space k are respectively given by

$$\delta\rho_\nu = \frac{1}{a^4} \int \frac{d^3q}{(2\pi)^3} \epsilon f_0(q) \Psi_0, \quad (14)$$

$$\delta P_\nu = \frac{1}{3a^4} \int \frac{d^3q}{(2\pi)^3} \frac{(1+\xi)q^2}{\epsilon} f_0(q) \Psi_0, \quad (15)$$

$$(\rho_\nu + P_\nu)\theta_\nu = \frac{k}{a^4} \int \frac{d^3q}{(2\pi)^3} \sqrt{1+\xi} q f_0(q) \Psi_1, \quad (16)$$

$$(\rho_\nu + P_\nu)\sigma_\nu = \frac{2}{3a^4} \int \frac{d^3q}{(2\pi)^3} \frac{(1+\xi)q^2}{\epsilon} f_0(q) \Psi_2, \quad (17)$$

where the perturbations Ψ_l satisfy the following Boltzmann equation

$$\begin{aligned} \dot{\Psi}_l + (1+\xi)\frac{q}{\epsilon}\frac{k}{2l+1}[(l+1)\Psi_{l+1} - l\Psi_{l-1}] \\ + \left(\delta_{2l}\frac{1}{15}\dot{h} + \delta_{2l}\frac{2}{5}\dot{\eta} - \delta_{0l}\frac{1}{6}\dot{h} \right) \frac{d\ln f_0}{d\ln q} = 0, \end{aligned} \quad (18)$$

in the synchronous gauge. Such a Boltzmann hierarchy is effectively truncated by adopting the following scheme [20]

$$\Psi_{l_{\max}+1} \approx \frac{(2l_{\max}+1)\epsilon}{(1+\xi)qk\tau} \Psi_{l_{\max}} - \Psi_{l_{\max}-1}. \quad (19)$$

The initial conditions for the perturbation Ψ_l and η on

super-horizon scales ($k\tau \ll 1$) are given by

$$\Psi_0 = -\frac{1}{4}\delta_\nu \frac{d \ln f_0}{d \ln q}, \quad (20)$$

$$\Psi_1 = -\frac{\epsilon}{3\sqrt{1+\xi} q k} \theta_\nu \frac{d \ln f_0}{d \ln q}, \quad (21)$$

$$\Psi_2 = -\frac{1}{2(1+\xi)} \left(\sigma_\nu + \frac{1}{4} \xi \delta_\nu \right) \frac{d \ln f_0}{d \ln q}, \quad (22)$$

$$\eta = 2C - \frac{5 + 9\sqrt{1+\xi} R_\nu - 5R_\nu}{6(15 + 4\sqrt{1+\xi} R_\nu)} C(k\tau)^2, \quad (23)$$

where

$$\delta_\nu = -\frac{2}{3} C(k\tau)^2, \quad (24)$$

$$\sigma_\nu = \frac{2(2 + R_\nu - \sqrt{1+\xi} R_\nu)}{3(15 + 4\sqrt{1+\xi} R_\nu)} C(k\tau)^2, \quad (25)$$

$$\theta_\nu = -\frac{(23 + 4R_\nu)\sqrt{1+\xi}}{18(15 + 4\sqrt{1+\xi} R_\nu)} C(k^4 \tau^3). \quad (26)$$

Here, C is a dimensionless constant determined by the amplitude of the fluctuations from inflation and $R_\nu = \rho_\nu/(\rho_\nu + \rho_\gamma)$ during radiation domination.

In order to compute the theoretical CMB power spectrum, we modified the Boltzmann CAMB code in [27] to appropriately incorporate the Lorentz-violating term in the neutrino sector. Actually the Lorentz-violating term affects not only the evolution of the cosmological background but also the behavior of the neutrino perturbations. From Eqs. (11)-(13), we see that increasing ξ decreases the number density, energy density and pressure of neutrinos, and thereby increases the redshift when the matter density equals the radiation density and reduces the expansion rate prior to and during the epoch of photon-baryon decoupling. This leads to reduced heights of the first and second peaks of the CMB while a nearly constant increase in the acoustic oscillation amplitudes at $l > 600$. On the other hand, the coefficient of the second term in the Boltzmann equation (18) plays an active role, which alters the shape of the CMB power spectrum caused by changing neutrino propagation. Decreasing ξ increases fluctuation power both at $l < 10$ and at $l > 100$. Moreover, the CMB is more sensitive to negative values of ξ than positive ones. These two effects can be distinguished from a change in the total mass of neutrinos or in the effective number of extra relativistic species [21–26], as shown in Figure 1.

IV. COSMOLOGICAL CONSTRAINTS

In our analysis we use a modified version of the publicly available CosmoMC package to explore the parameter space by means of Monte Carlo Markov chains technique [28]. We consider a flat Λ CDM models plus three Lorentz-violating neutrino species, described by a set of cosmological parameters

$$\{\Omega_b h^2, \Omega_c h^2, \Theta_s, \tau, n_s, A_s, \Sigma m, \xi\},$$

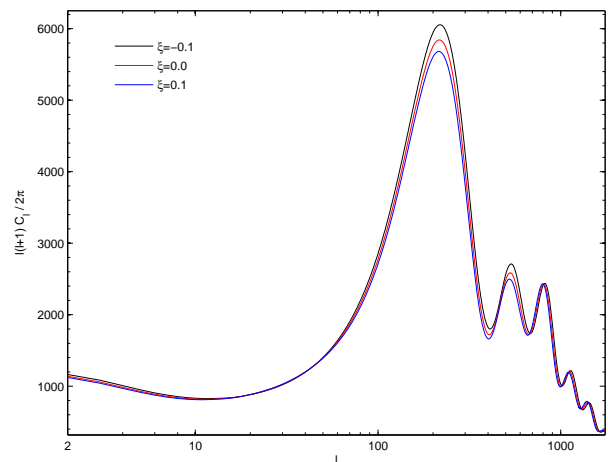


FIG. 1: Theoretical angular power spectrum of the CMB for $\xi = -0.1, 0, 0.1$. Here $\Sigma m = 3 \times 0.3$ eV is fixed.

where h is the dimensionless Hubble parameter such that $H_0 = 100h$ kms $^{-1}$ Mpc $^{-1}$, $\Omega_b h^2$ and $\Omega_c h^2$ are the physical baryon and dark matter densities relative to the critical density, Θ_s is the ratio of the sound horizon to the angular diameter distance at the photon decoupling, τ is the reionization optical depth, n_s and A_s are the spectral index and amplitude of the primordial curvature perturbations at the pivot scale $k_0 = 0.002$ Mpc $^{-1}$, Σm is the total mass of neutrinos assuming that the three neutrino masses are approximately degenerate, and ξ is the deformation parameter.

We use the seven-year WMAP (WMAP7) data with the likelihood code supplied by the WMAP team. We consider the Sunyaev-Zel'dovich (SZ) effect, in which CMB photons scatter off hot electrons in galaxy clusters. Given a SZ template, the effect is described by a SZ template amplitude A_{SZ} as in the WMAP paper [3]. We also impose Gaussian priors on the Hubble constant, $H_0 = 74.2 \pm 3.6$ kms $^{-1}$ Mpc $^{-1}$, measured from the magnitude-redshift relation of low- z type Ia supernovae [18], and on the distance ratios, $r_s/D_V(z = 0.2) = 0.1905 \pm 0.0061$ and $r_s/D_V(z = 0.35) = 0.1097 \pm 0.0036$, measured from BAO in the distribution of galaxies [17]. Here r_s is the comoving sound horizon size at the baryon drag epoch and D_V is the effective distance measure for angular diameter distance.

In Table I we summarize our results. With the data of WMAP7+ H_0 +BAO, the deformation parameter is estimated to be $\xi = -0.077 \pm 0.089$, which implies a null detection of Lorentz invariant violation within the error limits. Such large uncertainties in ξ mainly come from the strong correlation between the deformation parameter and the physical dark matter density as shown in Figure 2. In this case the uncertainties in $\Omega_c h^2$ are about three times larger than those derived in the standard Λ CDM model [3]. Since the deformation parameter is nearly uncorrelated with the total mass of neutrinos in

parameter	WMAP+ H_0 +BAO	WMAP+ H_0 +BAO	Planck+ H_0 +BAO
$100\Omega_b h^2$	2.312 ± 0.078	2.314 ± 0.081	2.260 ± 0.023
$\Omega_c h^2$	0.1200 ± 0.0103	0.1206 ± 0.0104	0.1133 ± 0.0033
Ω_Λ	0.715 ± 0.020	0.723 ± 0.017	0.725 ± 0.005
H_0	70.9 ± 2.4 km/s/Mpc	72.1 ± 2.2 km/s/Mpc	70.3 ± 1.0 km/s/Mpc
τ	0.089 ± 0.014	0.088 ± 0.014	0.088 ± 0.004
$\sum m$	< 0.51 eV	0 (fixed)	0 (fixed)
ξ	-0.077 ± 0.089	-0.073 ± 0.089	-0.005 ± 0.037

TABLE I: Mean values and marginalized 68% confidence level for the deformation parameter and other cosmological parameters. For the total mass of neutrinos, the 95% upper limit is given.

Figure 2, our constraints on the deformation parameter are not significantly changed if three neutrinos are massless, as we can see from Table I. Compared to particle physics experiments, cosmological observations yield much weaker constraints on the Lorentz-violation parameter in the neutrino sector. As listed in Table XIV of Ref. [29], previous constraints range from parts in 10^5 to parts in 10^{15} from time-of-flight measurements and various threshold analyses.

Moreover, we present the constraints on the deformation parameters from the ongoing Planck experiment [30] in Table I. Following the MCMC method described in Ref. [31], we generate synthetic data for the Planck experiment and then perform a systematic analysis on the simulated data. As we can see in Table I, the Planck data plus measurements of the Hubble constant and the angular diameter distance will reduce the uncertainties in ξ by a factor of 2.4. Therefore, Planck CMB measurements allow us to detect the signature of Lorentz invariant violation at 2σ confidence level if $|\xi| > 0.074$.

V. CONCLUSIONS

We studied the cosmological consequences of Lorentz-violating neutrinos. We obtained the generalized La-

grangian for a neutrino with the Coleman-Glashow type dispersion relation, and then derived its Boltzmann equation and initial conditions in the linear theory of cosmological perturbations. Using the 7-year WMAP data in combination with measurements of the Hubble constant and the BAO feature, we found no evidence for Lorentz invariant violation in the neutrino sector. The ongoing Planck experiment is expected to be able to give a more stringent constraints on the deformation parameter.

Acknowledgments

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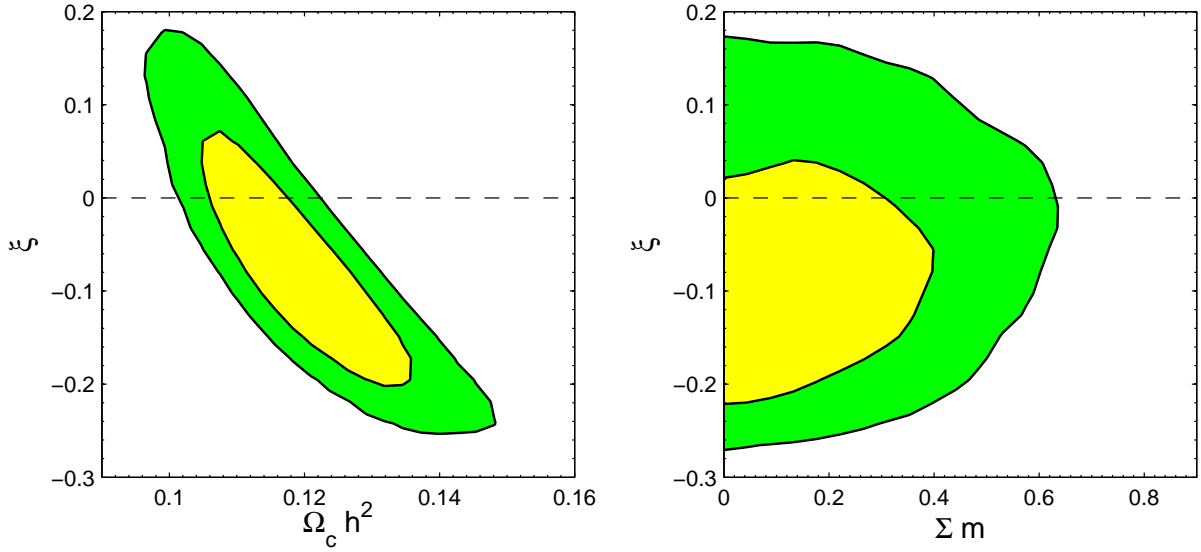


FIG. 2: Two-dimensional joint marginalized constraints (68% and 95% confidence level) on the deformation parameter ξ , physical dark matter density $\Omega_c h^2$ (left) and the total mass of neutrinos Σm (right), derived from the WMAP7+ H_0 +BAO data. The dashed line corresponds to Lorentz invariance.

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